# Topics in Algebra: $p$-Adic Numbers and Local Fields

Fall 2018 Syllabus

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Class Time: MWF 10:00 - 10:50
Office Hours: Tu 11 - 12, W 2 - 3

Course Website: http://math.rice.edu/~ae22/465_565F18.html

## Course Topics

The main topic we will be covering in this course is the field of $p$-adic numbers, with a focus on their applications to number theory. There are many ways to define the $p$-adic numbers, and each definition illuminates a different facet of this field. The first part of the course will focus on the analytic and topological structure of the $p$-adic numbers, although it is worth noting that it will be much more reminiscent of an algebra or elementary number theory course than an analysis or topology course. The second part will delve into roots of $p$-adic polynomials, and hence extensions of the $p$-adic numbers. The final part of the course will discuss local fields (which the $p$-adic numbers are an example of) and their properties in general. Time permitting, we will also touch on the main results of local class field theory.

## Text

The main text we will be using for this course is **$p$-Adic Numbers - An Introduction** by Fernando Gouvêa. This book will get us through roughly the first two thirds of the semester, along with supplementary materials which will be provided to you. For the remainder of the course we will be using notes which will be posted on the website and are freely available. For those who prefer to also have a textbook as reference, I would recommend either *Local Fields* by Jean-Pierre Serre or *Algebraic Number Theory* by Jürgen Neukirch. There is also a free textbook called *Local Fields and Their Extensions* by Fesenko and Vostokov which you can find online and from which I may pull some of my notes.

## Course Expectations and Homework

One of the reasons I have chosen to use Gouvêa's book for this course is because the exercises are interspersed with the reading material, the idea being that you complete the exercise before you continue reading. My expectation is that you will read or skim the sections corresponding to the previous lecture before the next one and do the exercises as you read.

Written homework will be due every two weeks and will be a selection of these exercises. The assignments will be posted well ahead of time so that you will know which exercises will be graded as you read those sections. While I strongly recommend you do all of the exercises, specifically the ones which are within the text rather than at the end of the section, only the selected exercises will be graded. The expectation is that you will be working on the homework throughout the two weeks and then will simply turn in your solutions to the selected problems.

## Grading Policy

Homework will count for 80% of your grade and the other 20% will be based on attendance and class participation.